§1.3 Incompressibility of Water.

Compressibility of Water

Actual Ease of Compression or Expansion
Depends on Physical Feature of Each Fluid Substance.

In the Case of Water?

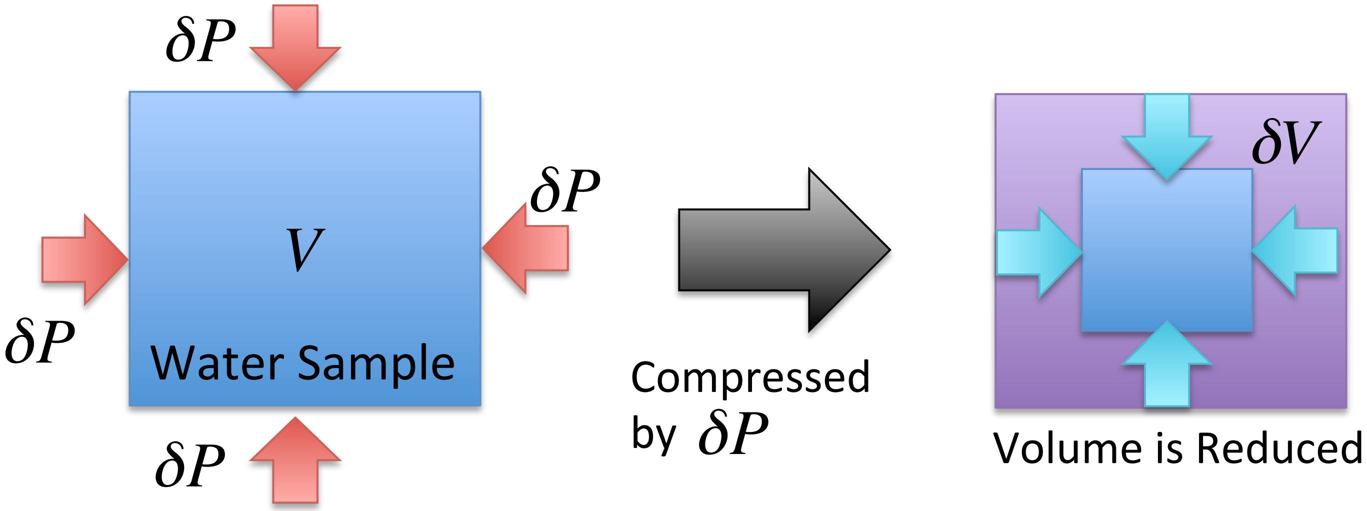
Ease of Compression is Investigated by Lab. Experiments.

・Indicator : Volume Compressibility $\kappa = -\frac{1}{V}\frac{\delta V}{\delta p}$

V :Volume of Water Sample

δP :Pressure Increasing

δV :Volume Chang due to δP



Physical Meaning of $\kappa = -\frac{1}{V}\frac{\delta V}{\delta p}$ → $\delta V = -\kappa V \delta p$

Assuming V =1m^3: $\delta V = -\kappa \delta p$

$\kappa \times \delta p$ gives "Volume Change" of V =1m^3 Water Sample
When Pressure increases δp

<u>Experimental Result for Water</u>

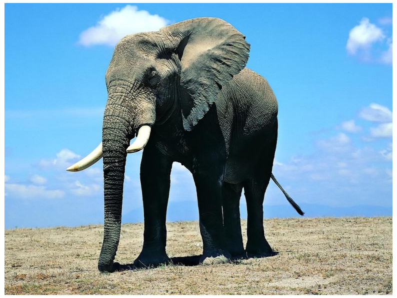
- κ depends on Water Temperature.
- Under Ordinary Water Temperature : T=20℃

$$\kappa = 0.00000000045[1/Pa] = 0.45 \times 10^{-9}[1/Pa]$$

Very Small Value! → Compression of Water: Very Difficult!

For Instance,

Elephant



Weight Reaches to 5,000[kg]

Water 1m³

$$\delta p = 5{,}000 \times 9.8[Pa] = 5 \times 10^4 [Pa]$$

Volume Change: $\delta V = -0.45 \times 10^{-10} \times 5 \times 10^4$

$$= 2 \times 10^{-6} [m^3]$$

Ratio of Volume Change: 0.0002[%]

Moai Statue

Easter Island

Weight Reaches to 90,000[kg]

Water 1m³

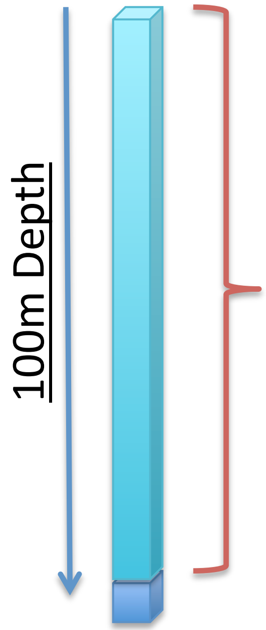
Ratio of Volume Change: 0.004[%]

In the Case of Actual Water Environmental,

Depth of Water Environmental in Land < 100m

(The Largest Water Reservoir in the World: Three Gorges Dam ・・・ 120～130m Depth)

Even in the Case of 100m Depth



Weight:
$100m \times 10^3 kg/m$
=100,000kg

≈

Weight:
90,000kg

Volume Change : 0.004[%]

In the Actual Water Environmental,

It is Reasonable that Volume of Water is assumed to be Constant.

We can Assume that Water is not Compressed and Expanded.

Volume Change of Fluids is Indicated by

Divergence of Velocity: $\nabla \cdot \mathbf{u} = \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}$

Volume of Fluid is

- Compressed : $\nabla \cdot \mathbf{u} < 0$
- Expanded: $\nabla \cdot \mathbf{u} > 0$
- Constant: $\nabla \cdot \mathbf{u} = 0$

If we Assume Volume of Water is Constant,

Divergence of Velocity must be $\nabla \cdot \mathbf{u} = 0$.

By using $\nabla \cdot \mathbf{u} = 0$, Conservative Equation can be simplified.

Original Conservative Eq.

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} + w\frac{\partial \phi}{\partial z} = -\phi\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$\nabla \cdot \mathbf{u} = 0$$

Simplified Conservative Eq.
under Water Incompressibility Assumption

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} + w\frac{\partial \phi}{\partial z} = 0$$

Usually, In the Hydraulics, This Eq. is Used for Material Transportation due to Flow.