§1.4 Derivation of Basic Set of Eqs. to be used in Environmental Flow.

Deriving a Basic Set of Fluid from Transport Eq.

By Regarding ϕ as Various Quantity, a Basic Set of Fluid Dynamics Eqs. is given.

1. $\phi = \rho$ (Density)

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = \frac{\partial}{\partial x}\left(\nu\frac{\partial \rho}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu\frac{\partial \rho}{\partial y}\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial \rho}{\partial z}\right)$$

2. $\phi = s$ (Salinity)

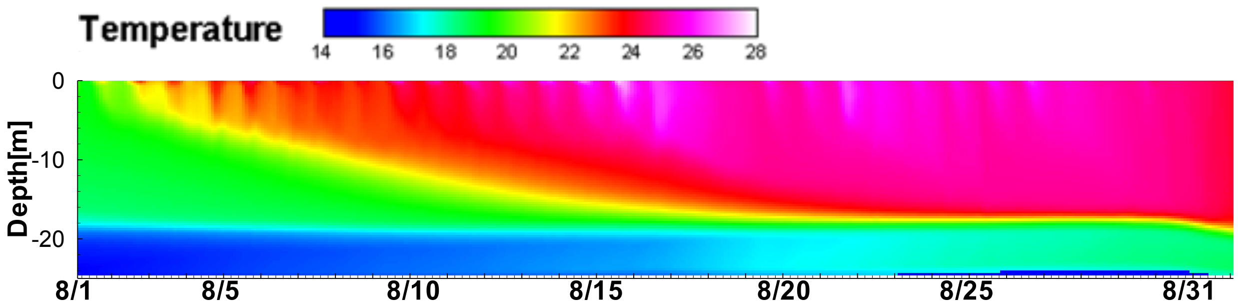
$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z} = \frac{\partial}{\partial x}\left(\nu\frac{\partial s}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu\frac{\partial s}{\partial y}\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial s}{\partial z}\right)$$

3. $\phi = T$ (Water Temperature)

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{\partial}{\partial x}\left(\nu\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu\frac{\partial T}{\partial z}\right)+\phi_{Heat}$$

ϕ_{Heat} : Energy Change Between Water Surface and Atmosphere.

Kamafusa Lake

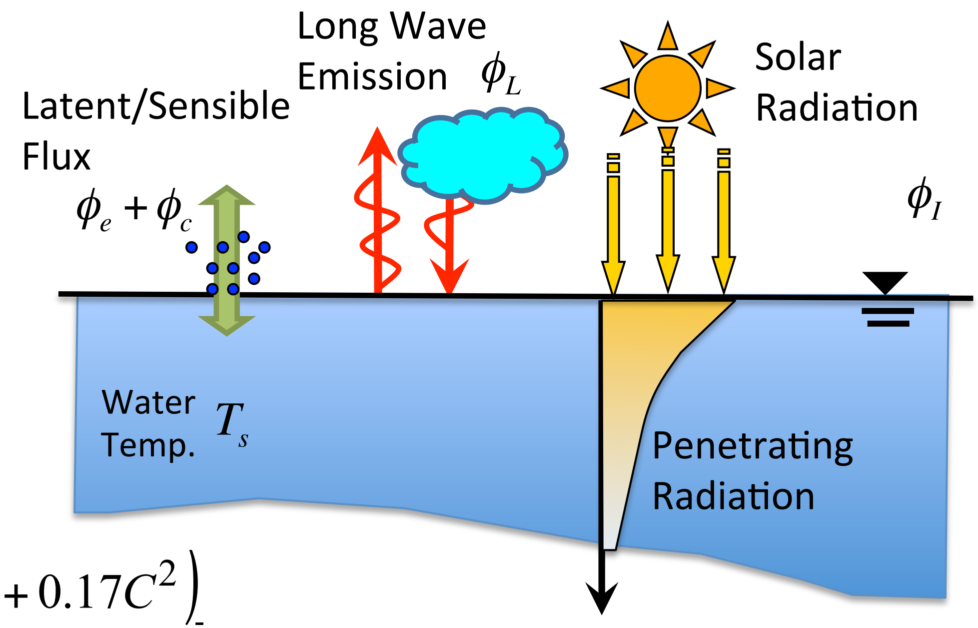


Time Change of Vertical Profile of T
<u>Summer Season</u>

Warm Water Layer is Generated Around Water Surface.

Energy Change Between Water Surface and Atmosphere.

ϕ_{Heat} : Evaluated with Meteorological Data (Air Temp., Humidity, Solar Radiation,…)



On Water Surface

$$\phi_{Heat} = (1-\alpha)\beta\phi_I - \phi_L - (\phi_e + \phi_c)$$

α: Reflection Rate (Albedo)

β: Absorption Rate

ϕ_L, ϕ_e, ϕ_c : Evaluated by Empirical Eq.

eg). $\phi_L = 0.97\sigma_s \left[T_s^4 - 0.937\times 10^{-5} T_A^6 (1.0 + 0.17C^2) \right]$

Under Water Surface

Penetrating Solar Radiation Decays Exponentially (Lambert-Beer's Law)

$$\phi_d = (1-r)(1-b_{abs})\phi_I \exp[-\eta \,|\, z-h \,|]$$

Deriving a Basic Set of Fluid from Transport Eq.

By Regarding ϕ as Various Quantity, a Basic Set of Fluid Dynamics Eqs. is given.

4. $\phi = \rho \mathbf{u} = \rho(u, v, w)$ (Momentum)

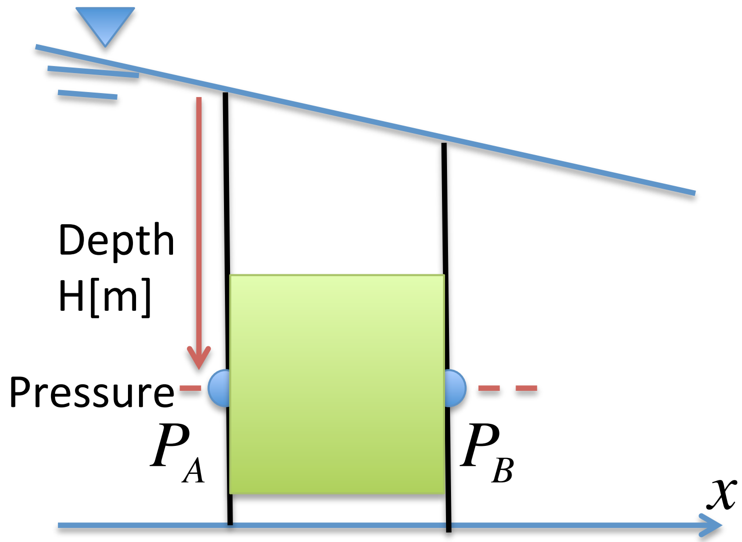
$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho u \frac{\partial \mathbf{u}}{\partial x} + \rho v \frac{\partial \mathbf{u}}{\partial y} + \rho w \frac{\partial \mathbf{u}}{\partial z} = \rho \frac{\partial}{\partial x}\left(\nu \frac{\partial \mathbf{u}}{\partial x}\right) + \rho \frac{\partial}{\partial y}\left(\nu \frac{\partial \mathbf{u}}{\partial y}\right) + \rho \frac{\partial}{\partial z}\left(\nu \frac{\partial \mathbf{u}}{\partial z}\right) + \mathbf{F}_E$$

In Actual Environment, Various Kinds of Extra Forces $\mathbf{F}_E$ are Affected.

$$\mathbf{F}_E = \mathbf{F}_P + \mathbf{F}_W + \mathbf{F}_B + \mathbf{F}_G$$

Pressure | Wind Friction | Bottom Friction | Gravity

Pressure

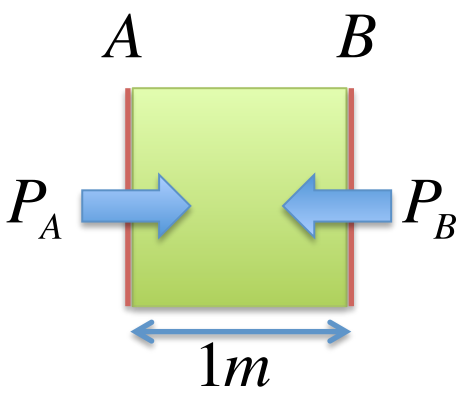


Hydraulic Static Pressure: $P = \rho g H$

$\propto$ Depth

If there is spatial change of Water Surface,

$$P_A \neq P_B$$



Water Element ($1\times1\times1=1m^3$)

Force On Surface A; P_A (+x Direction)

Force On Surface B; P_B (-x Direction)

Net Force Affect on Element;

$$F_P = (P_A - P_B) \quad = -\frac{\partial P}{\partial x}$$

Force due to Pressure

In 3D,

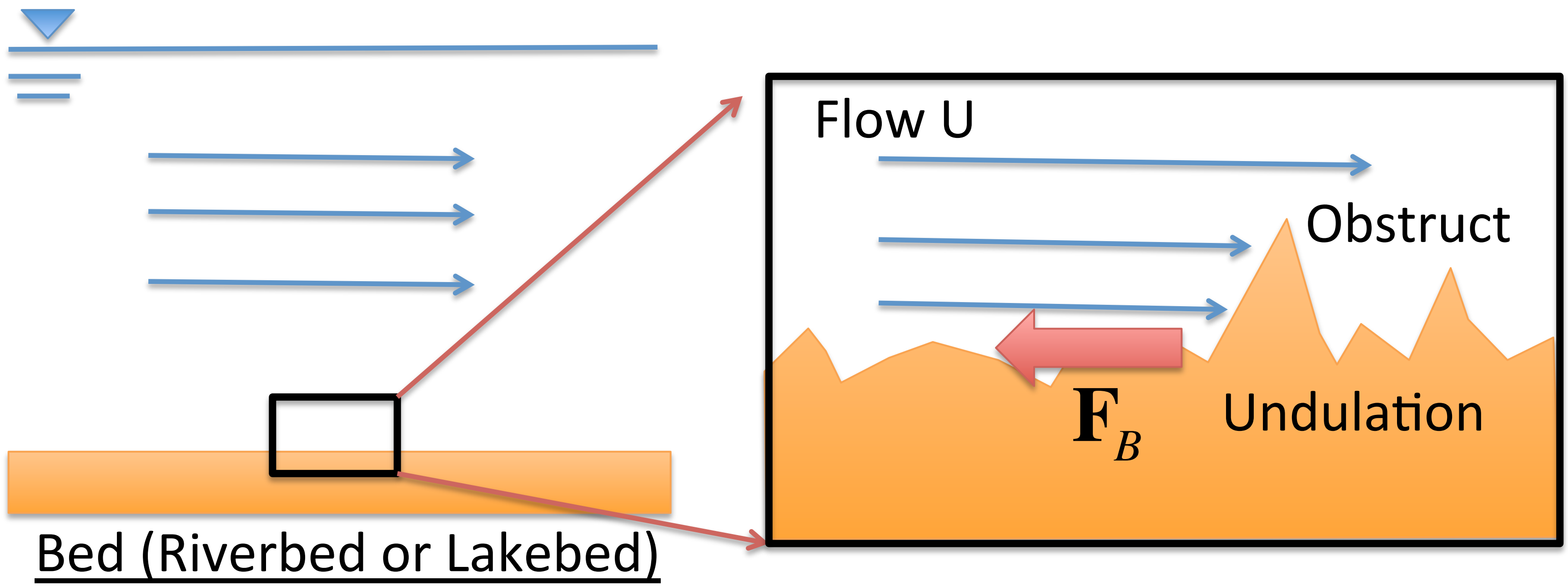
$$\mathbf{F}_P = -\begin{pmatrix} \dfrac{\partial P}{\partial x} \\ \dfrac{\partial P}{\partial y} \\ \dfrac{\partial P}{\partial z} \end{pmatrix} = -\nabla P$$

Pressure Force is Represented by Spatial Gradient of P in each Direction.

Bottom Friction

Even if the Bed Surface seems Flat,

In the micro scale, always there is Undulation.



Due to Undulation, Flow nearby the Bed is Obstructed.

→ Bottom Friction Force $\mathbf{F}_B$ Affects Near the Bed To Reduce the Flow U.

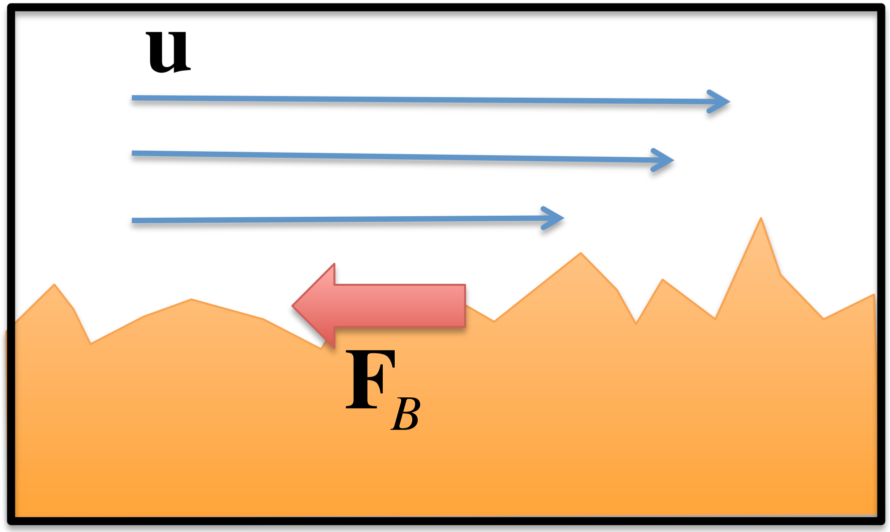
Bottom Friction

General Form of Friction Force; $\mathbf{F}_B = -\rho_{water} f_b |\mathbf{u}|\mathbf{u}$

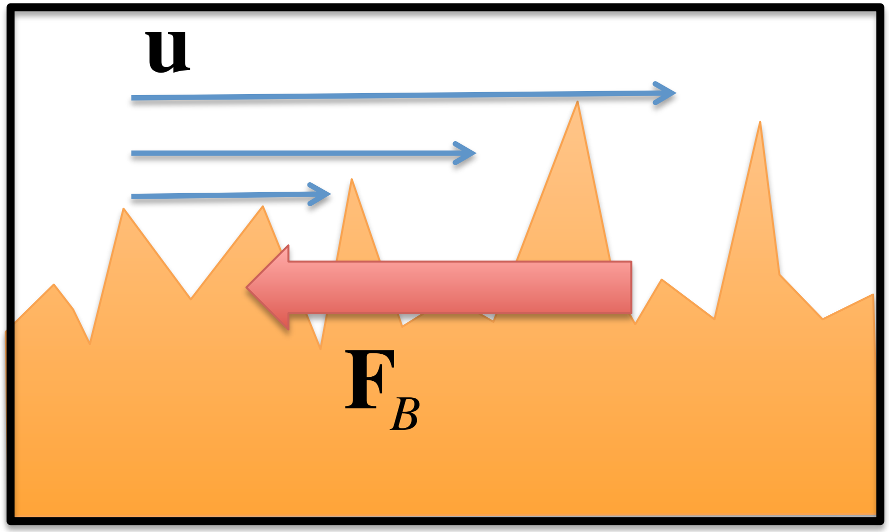
$\mathbf{u}$: Flow Velocity Near the Bed

f_b : Friction Coefficient (Magnitude of Friction)

Standard Value for River &Lake; $f_b = 0.0026$



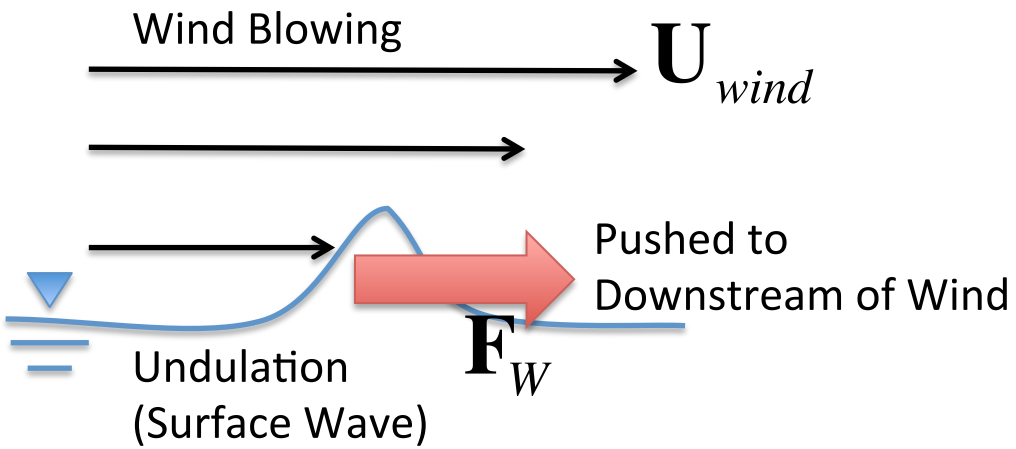
Small Undulation: Small Fric.



Large Undulation: Large Fric.

Friction Coefficient f_b Depends on Magnitude of Undulation.

Wind Friction



Wind Friction Force : $\mathbf{F}_W = \rho_{Air} C_D \left|\mathbf{U}_{wind}\right| \mathbf{U}_{wind}$

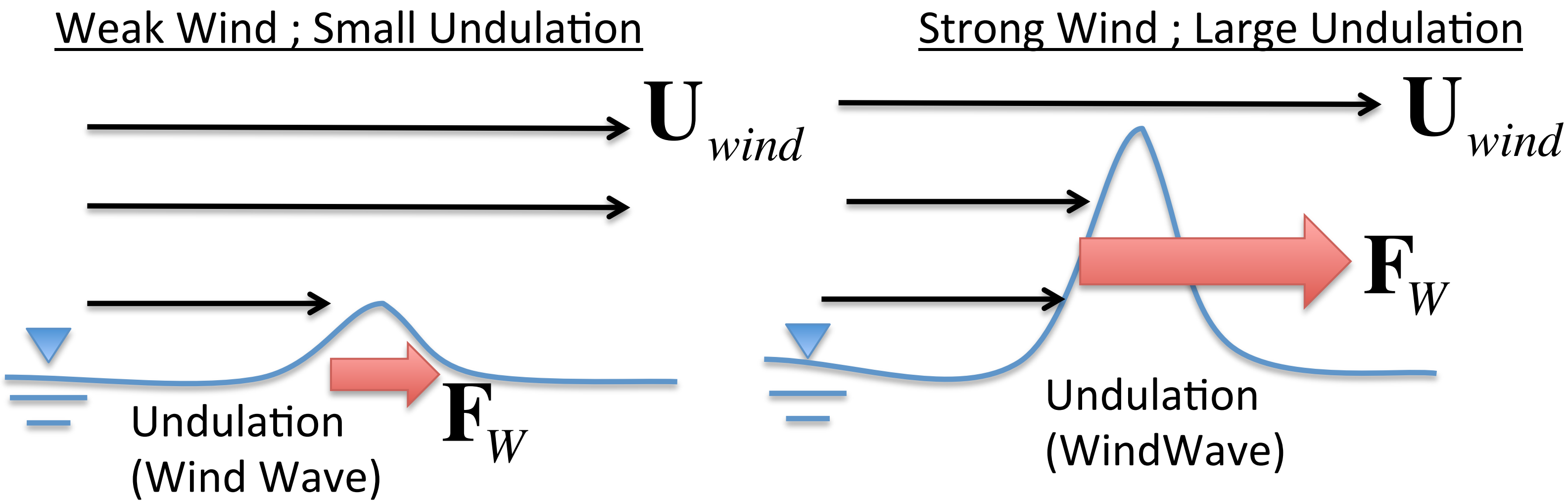
ρ_{Air} : Density of Air

$\mathbf{U}_{wind}$: Wind Velocity
(at the Height of 10m above from the water surface)

C_D :Wind Drag Coefficient (Efficiency of Energy Transf. from Wind)

Wind Friction

Cause of Undulation ;
"Wind Wave" Generated by
Strong Wind.



As Wind is Strong, Water accelerate more efficiently by Wind.

Generally, Cd is Assumed as a Function that Increases as Wind Speed.

$$C_D = 0.5 \times 10^{-3} \sqrt{\mathbf{U}_{wind}}$$

Equation of Motion for Water Flow

By Substituting Each Force,

$$\rho\frac{\partial \mathbf{u}}{\partial t}+\rho u\frac{\partial \mathbf{u}}{\partial x}+\rho v\frac{\partial \mathbf{u}}{\partial y}+\rho w\frac{\partial \mathbf{u}}{\partial z}=-\nabla P-\rho f_b\left|\mathbf{u}\right|\mathbf{u}+\rho_{Air}C_D\left|\mathbf{U}_{wind}\right|\mathbf{U}_{wind}$$

$$+\rho\frac{\partial}{\partial x}\left(\nu\frac{\partial \mathbf{u}}{\partial x}\right)+\rho\frac{\partial}{\partial y}\left(\nu\frac{\partial \mathbf{u}}{\partial y}\right)+\rho\frac{\partial}{\partial z}\left(\nu\frac{\partial \mathbf{u}}{\partial z}\right)+\mathbf{F}_G$$

Gravity Force : $\mathbf{F}_G=\begin{pmatrix}0\\0\\-g\rho\end{pmatrix}$

Navier-Stokes Eq. for Environmental Flow

$$\frac{\partial \mathbf{u}}{\partial t} + u\frac{\partial \mathbf{u}}{\partial x} + v\frac{\partial \mathbf{u}}{\partial y} + w\frac{\partial \mathbf{u}}{\partial z} = -\frac{1}{\rho}\nabla P - f_b\left|\mathbf{u}\right|\mathbf{u} + \frac{\rho_{Air}}{\rho}C_D\left|\mathbf{U}_{wind}\right|\mathbf{U}_{wind}$$

$$+\frac{\partial}{\partial x}\left(\nu\frac{\partial \mathbf{u}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu\frac{\partial \mathbf{u}}{\partial y}\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial \mathbf{u}}{\partial z}\right) - \mathbf{g}$$

$$\mathbf{g} = \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}$$

Summary of Basic Equations for Water Flow

Unknown Physical Quantity to be Solved to Know the Flow; 5 Quantity (u, v, w, ρ, P)

Momentum Eq.(3 Eqs.)

$$\frac{\partial \mathbf{u}}{\partial t} + u\frac{\partial \mathbf{u}}{\partial x} + v\frac{\partial \mathbf{u}}{\partial y} + w\frac{\partial \mathbf{u}}{\partial z} = -\frac{1}{\rho}\nabla P - f_b\left|\mathbf{u}\right|\mathbf{u} + \frac{\rho_{Air}}{\rho}C_D\left|\mathbf{U}_{wind}\right|\mathbf{U}_{wind}$$

$$+\frac{\partial}{\partial x}\left(\nu\frac{\partial \mathbf{u}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu\frac{\partial \mathbf{u}}{\partial y}\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial \mathbf{u}}{\partial z}\right) - \mathbf{g}$$

Transport Eq. for Density

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = \frac{\partial}{\partial x}\left(\nu\frac{\partial \rho}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu\frac{\partial \rho}{\partial y}\right) + \frac{\partial}{\partial z}\left(\nu\frac{\partial \rho}{\partial z}\right)$$

Incompressibility

$$\nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

According to Above 5 Eqs., We can know 5 Quantity (u, v, w, ρ, P)