§2 Features of Flow caused by Spatial Difference of Water Density.

§2.1 Origin of Water Density Change

Density of Environmental Water

Density of Pure Water: $\rho_{Pure} \cong 1{,}000[kg/m^3]$

In Natural Water Environment,

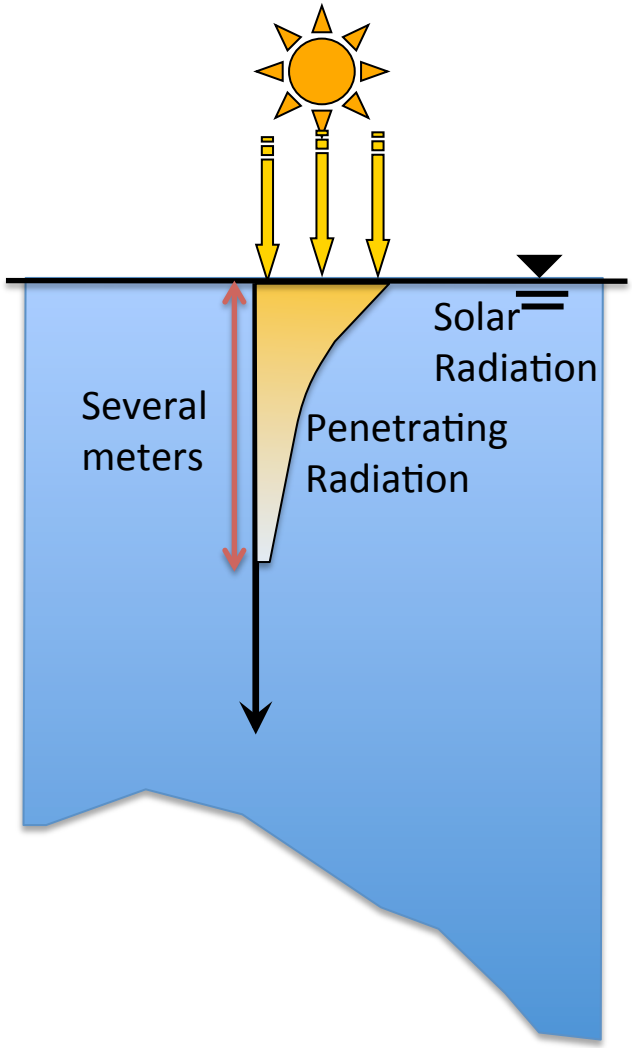
Density of Water can Change According to Water Temperature
and Amount of Dissolved Substances.
(Salinity, Suspended Sediment,….)

⇨ Density of Water in Natural Environment Fluctuates in Space & Time.

Ex) Lake in Summer Season

> High Air Temperature

> Strong Solar Radiation (Light) is Irradiated



> Solar Radiation can Penetrate Water Downward,

But, Decaying Exponentially with Depth.

$$\phi \propto \exp[-"Depth"/L]$$

L : Depth Solar Radiation can Penetrate
(Lakes ~5m)

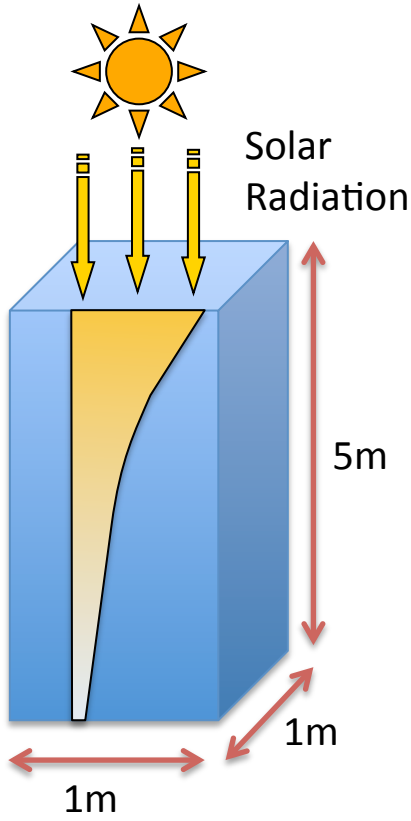
※ Energy is mainly Deposited Around the Surface.

※ Water Near the Surface is Heated Selectively.

eg). Peak Energy Deposition

Tokyo (Summer) ～10 MJ/m^2/day

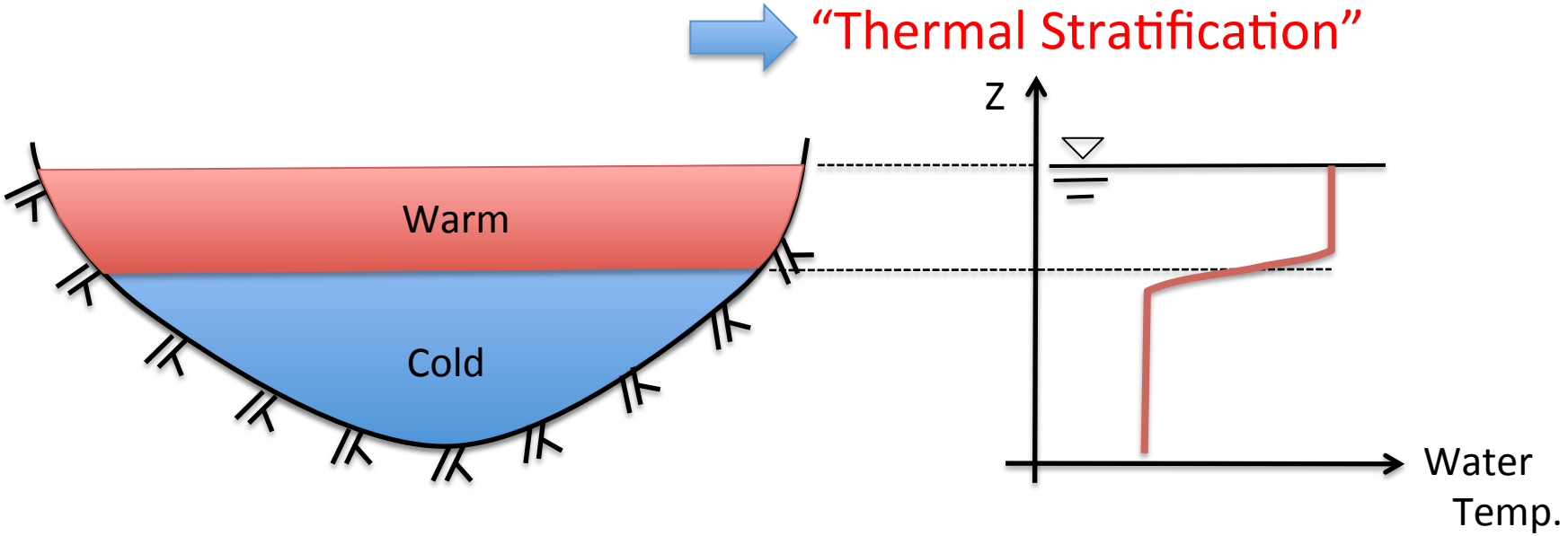
Water Column of 5m Height
(Volume in That Radiation can Penetrate)



<u>Rising of Water Temp. Due to Radiation;</u>

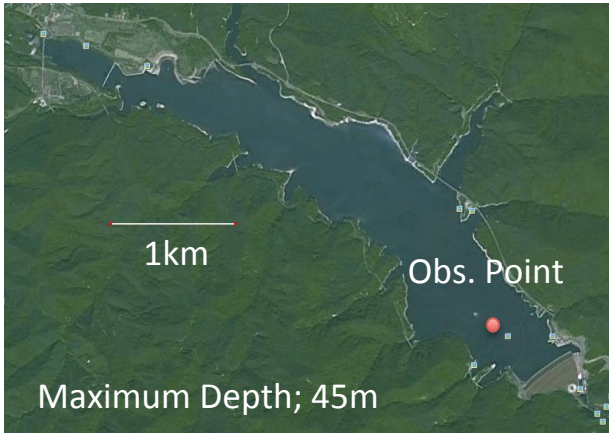
10 MJ/m^2/day ⇒ ΔT～0.5 [℃/day]

In Lakes, Warm Water Layer is Generated around the Water Surface in Summer.

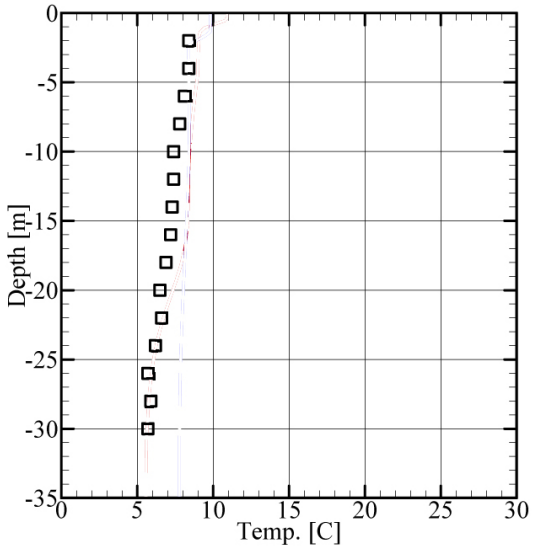


Observed Vertical Profile

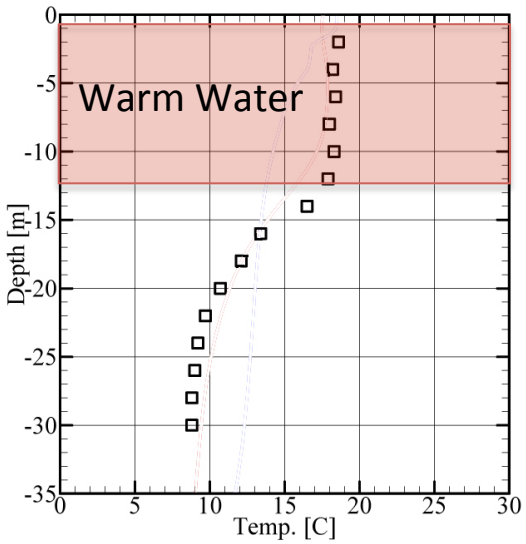
Shichigashuku Lake



Winter Season



Summer Season



Density Change Due to Water Temperature

Warm Water is Lighter than Cold Water

Empirical Relation ; $\rho = \rho_0 \{1 - \beta \times (T - 3.98)\}$

T : Water Temperature [℃]

ρ_0 : Density of Water at T =3.98 [℃]

β : Volume Expansion Coefficient [1/℃]

Value of β Slightly Depends on Water Temperature,

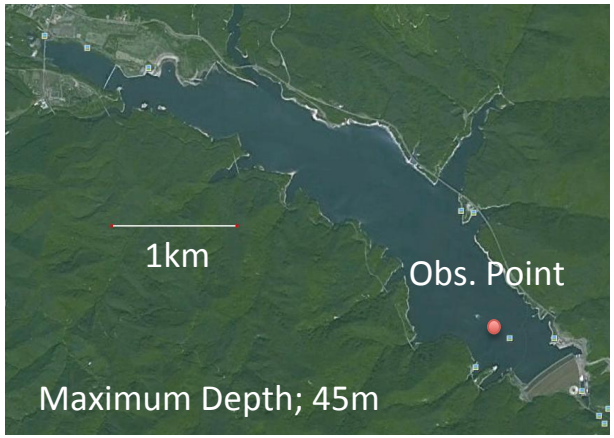
T～20[℃] ⇒ β～0.0002 [1/℃]

According to Empirical Relation,

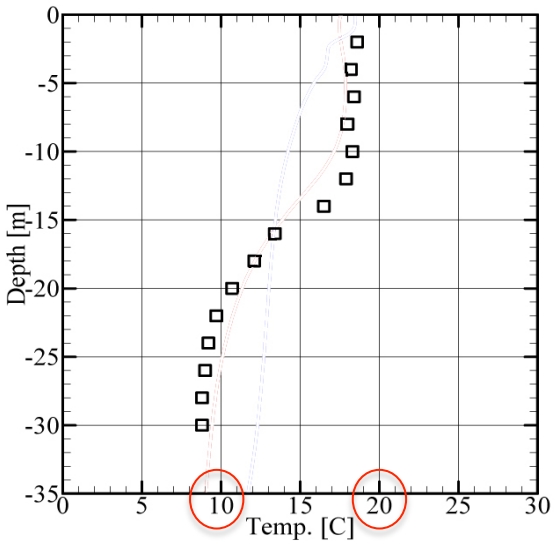
Density Becomes 0.2 [kg/m3] Lighter per 1 [℃] Increasing.

Actual Lake,

Shichigashuku Lake



Summer Season



Summer Season ; Surface Warm Layer ～ 20 [℃]

Bottom Cold Layer ～ 10[℃]

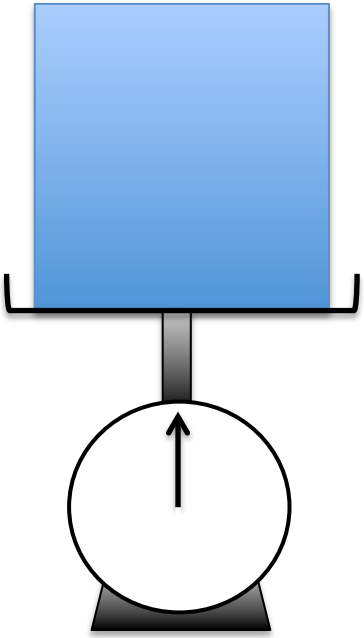
Density Difference Between Surface and Bottom Layer; $\Delta\rho \cong 2.0[kg/m^3]$

Density Change Due to Dissolved Substances

>Many Kind of Substances are Easily Dissolved in Water.

eg) Sea Water ; Salt (NaCl)

>Water Becomes More Heavy Due to Weight of Dissolved Substances.



Empirical Relation for Saline Water

$$\rho = \rho_{Fresh}\left(1 + 0.77 \times 10^{-3} \times S\right)$$

$\rho_{Fresh} \cong 1{,}000[kg/m^3]$: Density of Fresh Water (T =10[℃])

S [‰] : Salinity (Weight (g) of Dissolved Salts (NaCl) per 1 kg Water)

Salinity of Sea Water ; S =35 [‰]

Density of Sea Water ; $\rho_{Sea} = \rho_{Fresh}\left(1+0.77\times10^{-3}\times35\right) = \rho_{Fresh} + \rho_{Fresh}\times10^{-3}\times28$

Increasing of Density ; $\Delta\rho = \rho_{Sea} - \rho_{Fresh} = \rho_{Fresh}\times10^{-3}\times28 = 28[kg/m^3]$

$\because \rho_{Pure} \cong 1{,}000[kg/m^3]$

※ Per 1 m^3 Volume, Sea Water is Almost 30 kg Heavier than Fresh Water.

cf. Typical Density Difference due to Water Temp. in Lakes;

$$\Delta\rho \cong 1.0[kg/m^3]$$

Salinity Has Larger Impact on Density Change than Water Temp.

Density Change Due to Suspended Sediment

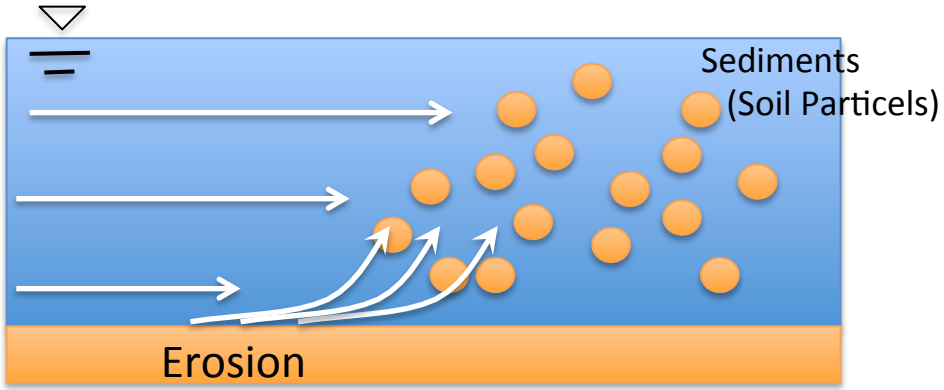
River Flowing through Arid Region.

Mountainous River After Heavy Rain.

When
- River Flows over Arid Region
- Flow Becomes Massive After Rain Fall

→ Flow Erodes River Bed.

Water of River is Colored in Muddy & Contains Many Suspended Soil Particles (Sediments).

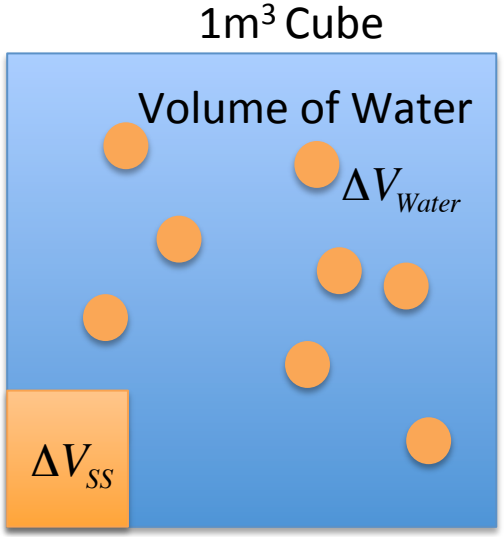


Density Change Due to Suspended Sediment

> Density of Each Sediment Particle($\rho_{SS} = 2{,}500[kg/m^3]$;Sand)
is More Heavy than Water($\rho_{Water} \approx 1{,}000[kg/m^3]$).

> Density of Water Containing Suspended Sediments (SS)

➡ More Heavy than Pure Water.



Evaluation of Water Density Containing SS

> How Much SS is Contained ; Measured as "Turbidity T_u".

Turbidity $T_u\,[\mathrm{kg}/m^3]$; Weight of SS (kg) in $1m^3$ of Water.

> Volume of SS in 1m³ Cube; $\Delta V_{SS} = \dfrac{T_u}{\rho_{SS}}[m^3]$

> Remaining is Volume of Water ; $\Delta V_{Water} = \left(1 - \Delta V_{SS}\right)[m^3]$

Density of Water Containing SS is Evaluated by Sum-up of Each Weight

$$\rho_{ContainSS} = \left(\rho_{SS} \times \Delta V_{SS} + \rho_{Water} \times \Delta V_{Water}\right)$$

$$\therefore \rho_{ContainSS} = \rho_{Water}\left\{1 + T_u\left(\frac{1}{\rho_{Water}} - \frac{1}{\rho_{SS}}\right)\right\}$$

Density Change Due to Suspended Sediment

eg) the Yellow River; $T_u \approx 76.6[kg/m^3]$

Pure Water Density; $\rho_{Water} = 1{,}000 = 10^3[kg/m^3]$

SS Density (Sand); $\rho_{SS} = 2.5 \times 10^3[kg/m^3]$


The Yellow River (Qinghai, China)

$$\rho_{ContainSS} = \rho_{Water}\left\{1 + T_u\left(\frac{1}{\rho_{Water}} - \frac{1}{\rho_{SS}}\right)\right\}$$

$$= 10^3 \times \left\{1 + 76.6\left(\frac{1}{10^3} - \frac{1}{2.5 \times 10^3}\right)\right\}$$

$$= 10^3 \times \left\{1 + 76.6 \times 10^{-3} \times (1 - 0.4)\right\}$$

$$\therefore \rho_{ContainSS} \cong 1045[kg/m^3]$$

※ Per 1 m^3 Volume, Water of the Yellow River is Almost 45 kg Heavier than Fresh Water.

Summary of Causes to Change Water Density

>In Actual Natural Water Environment, Density of Water can Change.

>Three Main Causes;

Order of Weight-Increasing per $1m^3$

- Water Temperature → Several kg
- Dissolved Substances (Salt) → Several tens kg
- Suspended Sediments (Soil Particles) → Several tens kg