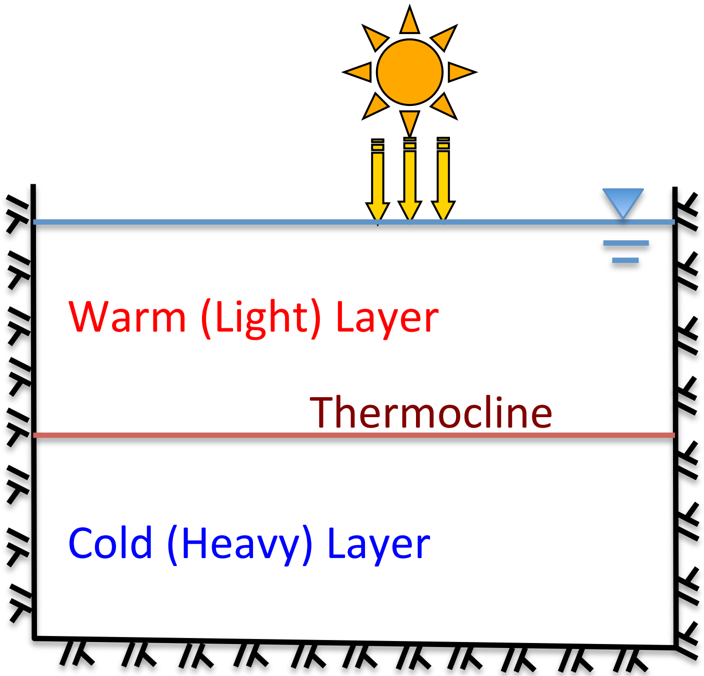
§2.3 Feature of Density Stratification in Steady State – Behavior of Interface

Static Density Stratification

In the Closed Water Environments (Lakes, Bays,..), Water Flow tends to be Weak.

→ Stable Density Stratification Exists.



In the Calm Lakes

> Two Water Layers are Generated by Solar Radiation.

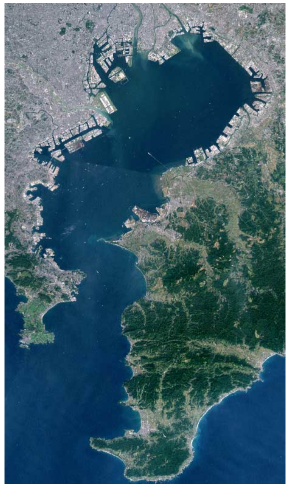
> Interface between Two Layers, on That Water Temperature Changes, is Named "Thermocline".

In the Closed Bay

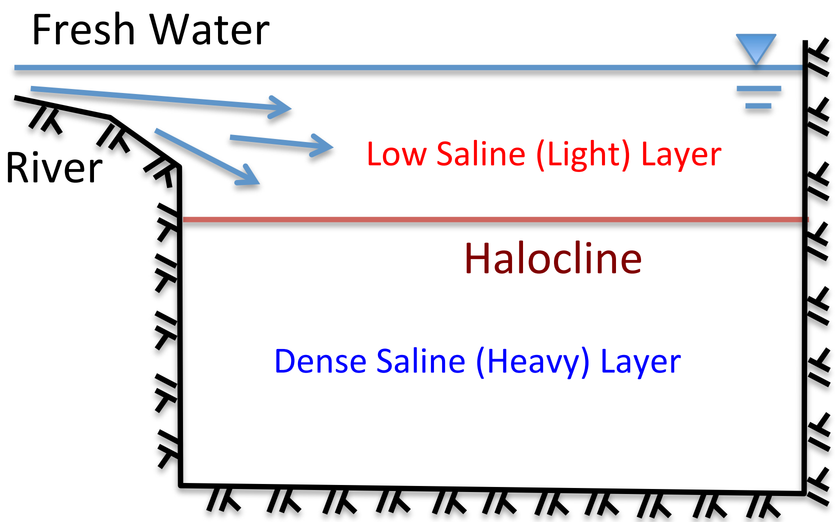
> Fresh Water Empties From Rivers, Upper Sea Water is Diluted.

> Generate Upper Low Saline Layer that is Lighter than Sea Water.

> Interface is Named "Halocline".



Tokyo Bay

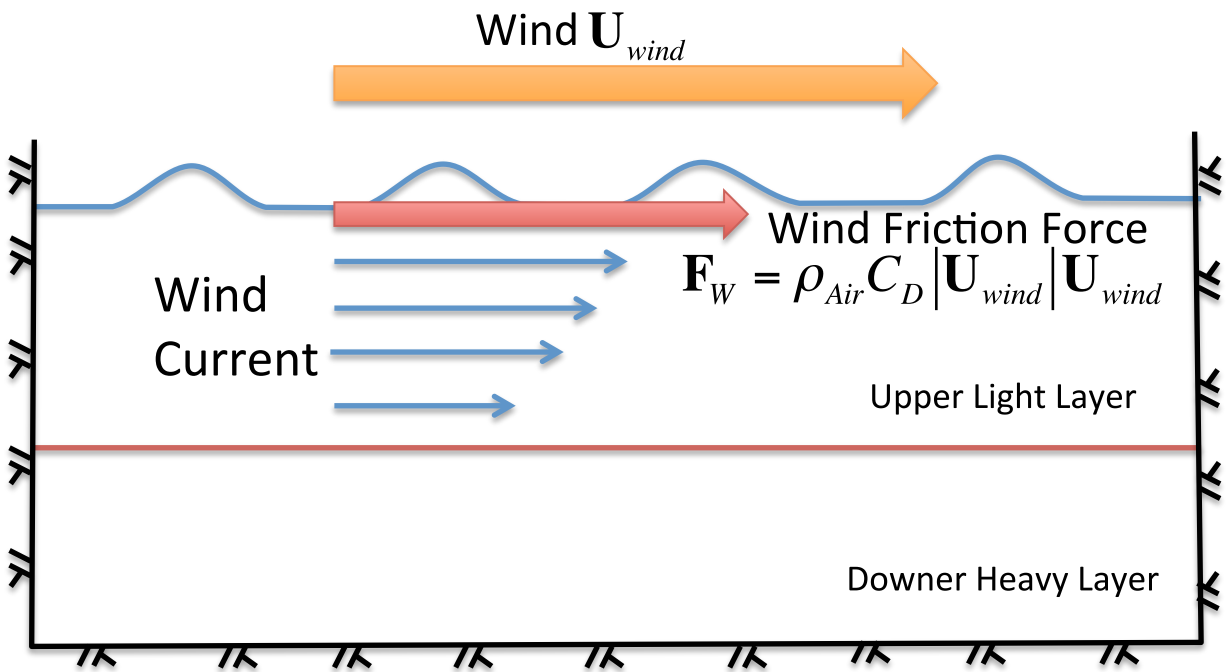


If there is no Extrafoces (Wind Friction, …),

Interface (Thermocline, Halocline) is Kept to be Flat.

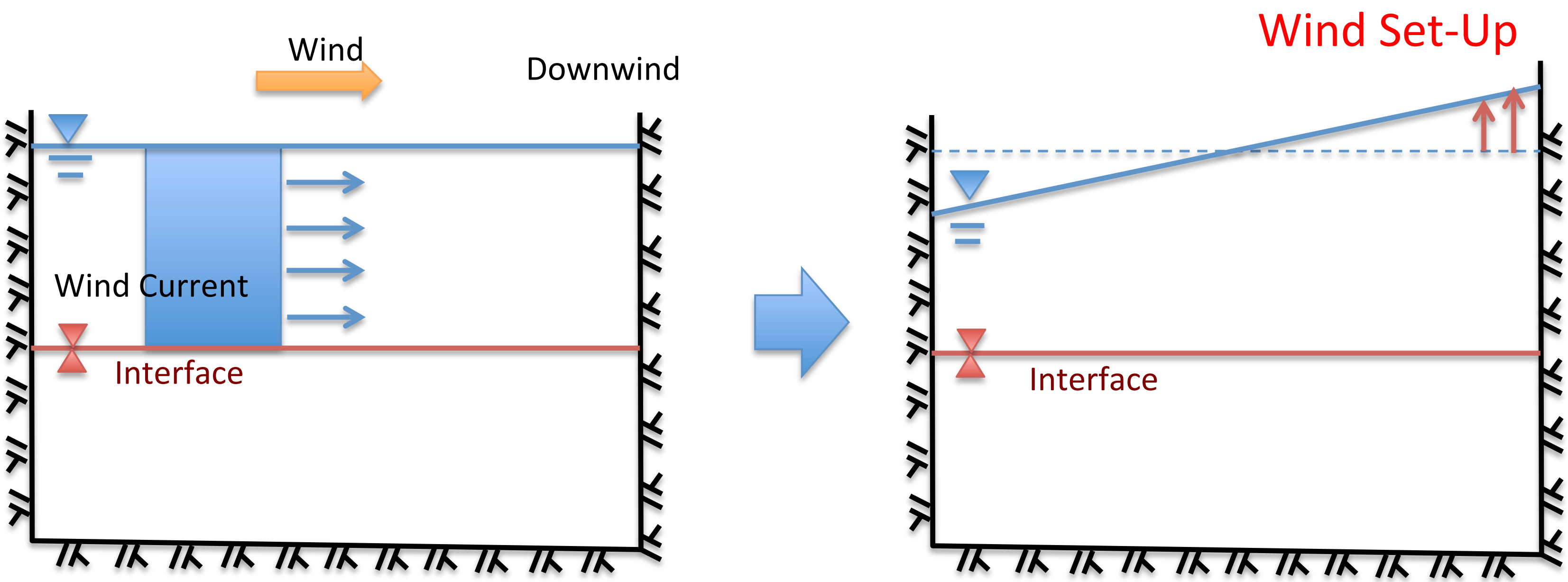
Wind Blowing on Water Surface

Affects “Wind Stress (Friction)” Force on the Upper Water Layer.



Upper Light Water Layer is Dragged to Downwind Direction
And Wind Current Flowing to Downwind is Generated.

When a Strong Wind is Blowing Continuously,



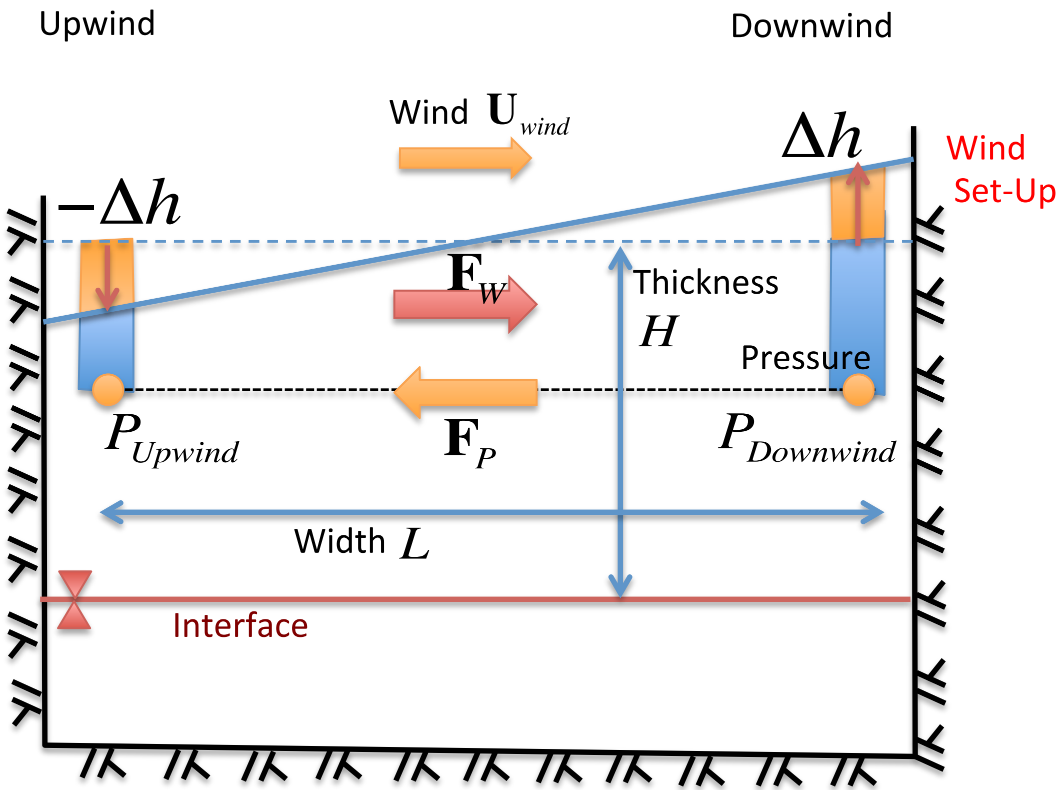
Upper Light Water is Swept to Downwind and is Accumulated in Downwind Area.

➡ Water Level (Water Surface) Rises Up in Downwind Area.

"Wind Set-Up" Effect.

Rough Evaluation of Wind Set-Up

>Rising due to Wind Set-Up can be Roughly Evaluated from a Balance of Forces.



Wind Friction Force;

$$\mathbf{F}_W = \rho_{Air} C_D |\mathbf{U}_{wind}| \mathbf{U}_{wind}$$

$$C_D = 0.5 \times 10^{-3} \, | \mathbf{U}_{Wind} |$$

Hydrostatic Pressure Force;

in Downwind
- Because Weight of Overhead Water Increases; $\Delta h \times \rho_{Water}$
- Pressure Increases; $P_{Downwind} = P_{Initial} + \Delta h \times g \times \rho_{Water}$

in Upwind
- Because Weight of Overhead Water Decreases; $-\Delta h \times \rho_{Water}$
- Pressure Decreases; $P_{Upwind} = P_{Initial} + \Delta h \times g \times \rho_{Water}$

Hydrostatic Pressure Force; $\mathbf{F}_P \cong H \times \nabla P = H \times \dfrac{\partial P}{\partial x} = H \times \dfrac{P_{Downwind} - P_{Upwind}}{L} \approx \dfrac{H}{L} \rho_{Water} g \Delta h$

L : Width of Water Area $\quad$ H : Thickness of Upper Water Layer

In the Steady State

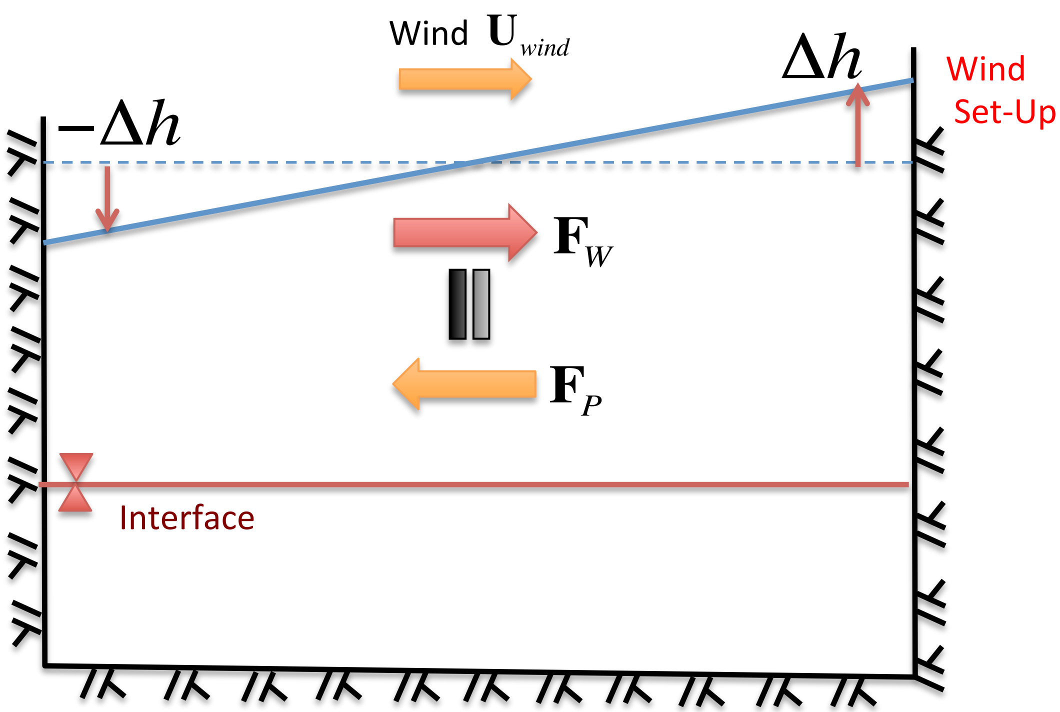
> Forces Must be Balance.

$$\mathbf{F}_W = \mathbf{F}_P$$

> Substituting Explicit Forms,

$$\mathbf{F}_W = \frac{H}{L}\rho_{Water} g \Delta h$$

$$\therefore \Delta h = \frac{L}{Hg\rho_{Water}}\mathbf{F}_W$$



> When Wind Speed is 10m/s,

$$C_D = 0.5\times10^{-3} |\mathbf{U}_{Wind}| = 0.5\times10^{-3}\times10 = 0.005$$

$$\rho_{Air} \cong 1.3[kg/m^3]$$

$$\mathbf{F}_W = \rho_{Air} C_D \left|\mathbf{U}_{wind}\right| \mathbf{U}_{wind} = 1.3\times0.005\times10\times10 = 0.65[kg/s^2m]$$

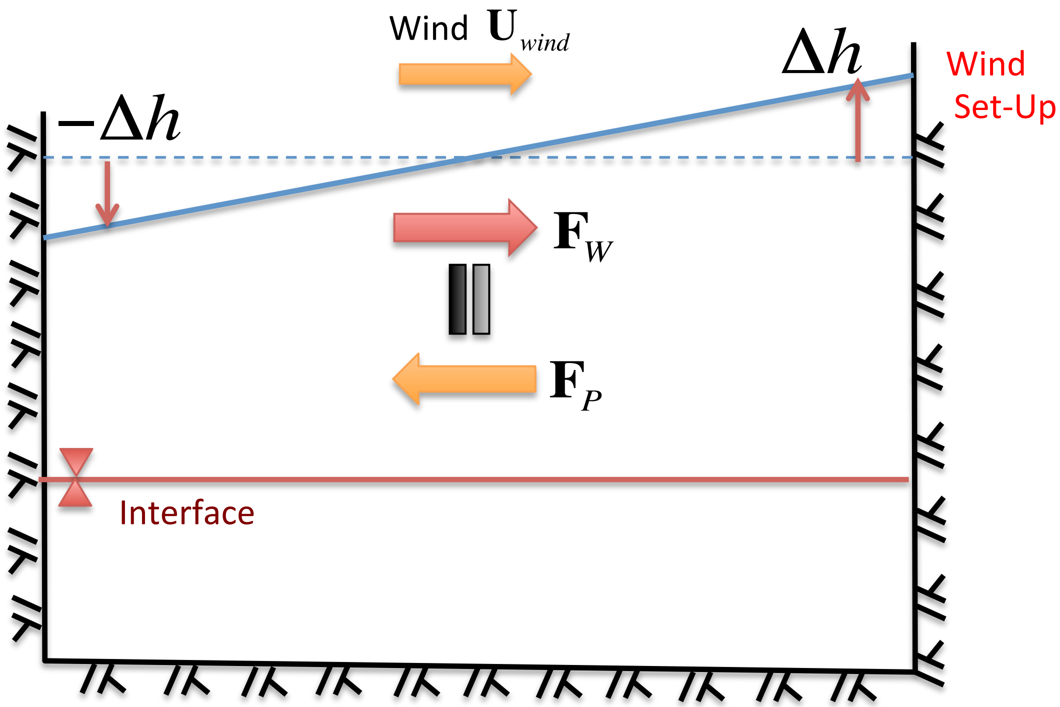
Assuming Typical Scale of Lakes,

Width: $L \approx 10km = 10^4 m$

Depth: $H \approx 10m$

Rising of Water Level

$$\Delta h = \frac{L}{Hg\rho_{Water}} \mathbf{F}_W$$

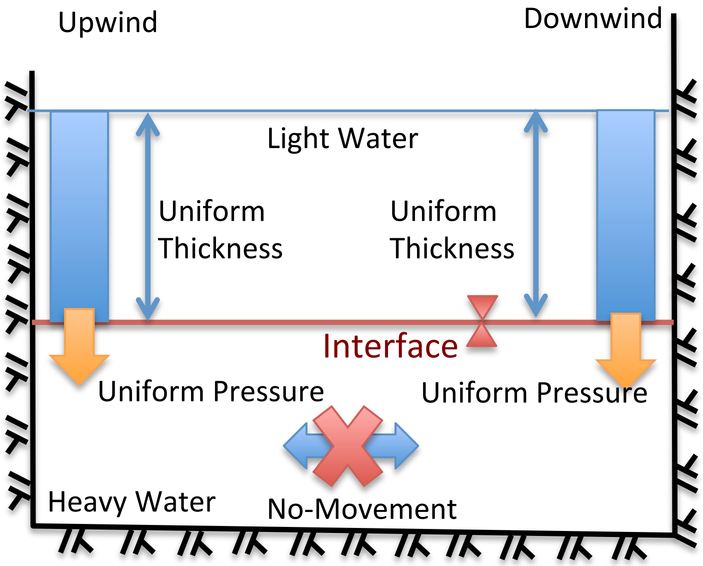


$$\Delta h = \frac{10^4}{10 \times 9.8 \times 10^3} \times 0.65 \cong 0.65 \times 10^{-1} = 6.5[cm]$$

When 10m/s Wind Blows in Typical Lakes,
Water Level in Downwind can Rise up Several [cm]

Turn Back to Consideration of Density Stratification

Bottom Heavy Water is also Affected by Surface Rising & Interface Changes.



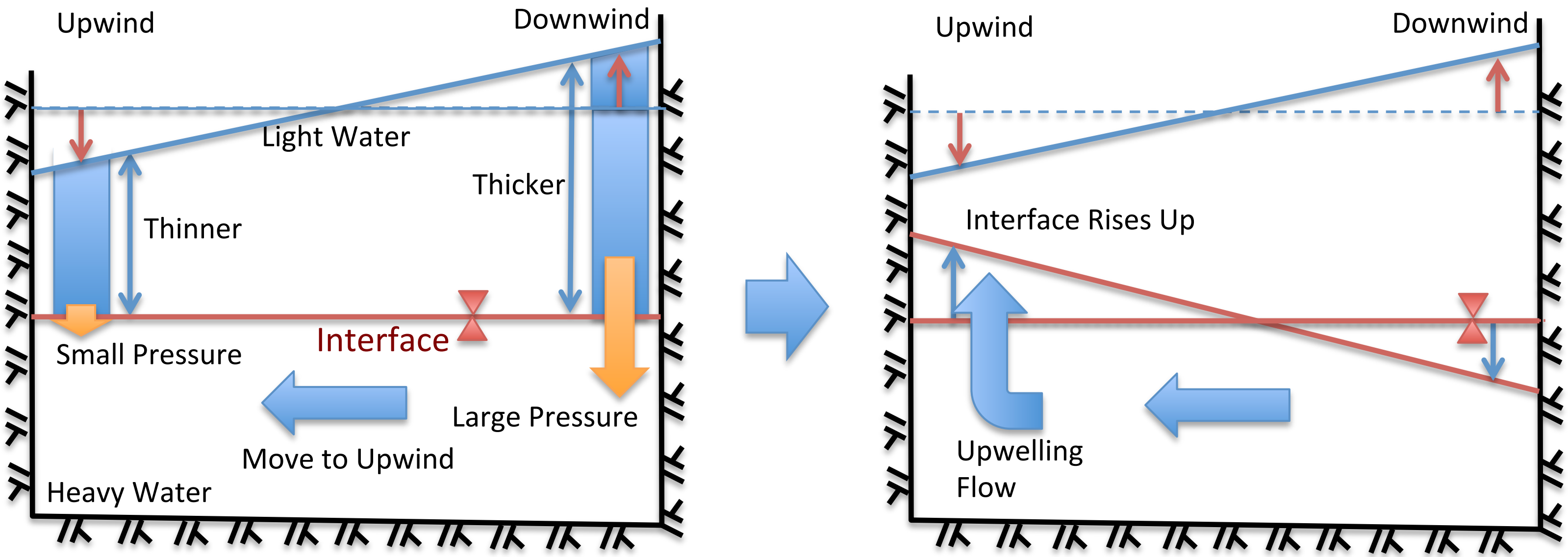
In the Case of Flat Water Surface,

> Thickness of Upper Light Layer is Spatially Uniform.

> Pressure Pushing Down on Interface is Uniform.

> In Bottom Layer, No-Movement is Generated.

Turn Back to Consideration of Density Stratification



In the Case of Non-Flat Water Surface,

> Thickness of Upper Light Layer is { Thicker in Downwind / Thinner in Upwind

> Pressure Pushing Down on Interface is { Larger in Downwind / Smaller in Upwind

> In Bottom Layer, Water is Pushed to Upwind & Flow to Upwind is Generated.

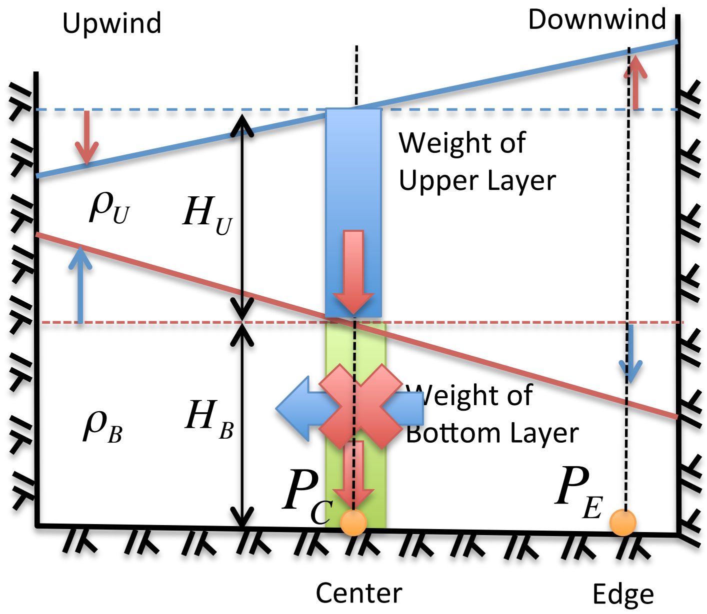
In Upwind Area , Upwelling Flow is Generated & Interface Rise Up

Maximum Rising Height of Bottom Layer

>When Wind is Blowing Continuously,

>After a Long Time Passing,

Rising up of Interface Reach a Steady State
&
Movement of Bottom Layer will Stops.



In the Steady State,

>There No Horizontal Movement.

>Hydrostatic Pressure on the Bottom Surface Must be Uniform.

"Pressure at Center P_C" = "Pressure at Edge P_E"

>Pressure at Center, P_C = "Weight of Upper Layer" + "Weight of Bottom Layer"

$$P_C = "H_U \times g \times \rho_U" + "H_B \times g \times \rho_B"$$

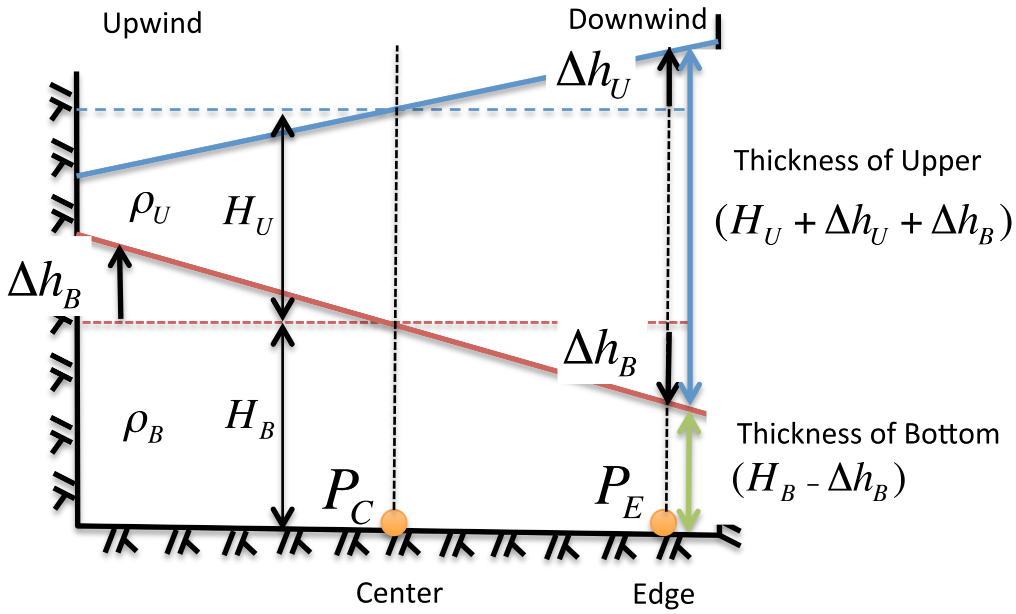
H_U : Thickness of Upper Layer H_B : Thickness of Bottom Layer

ρ_U : Density of Upper Layer ρ_B : Density of Bottom Layer

Maximum Rising Height of Bottom Layer

Δh_U : Rising Height of Water Level

Δh_B : Rising Height of Interface



> Thickness at “Edge”;

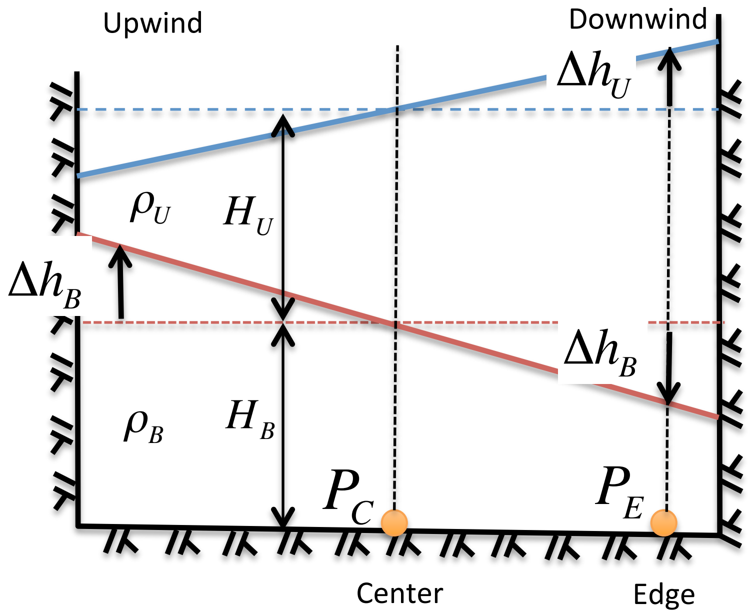
Upper Layer : $(H_U + \Delta h_U + \Delta h_B)$

Bottom Layer : $(H_B - \Delta h_B)$

>Pressure at Edge,

$$P_B = "\left(H_U + \Delta h_U + \Delta h_B\right) \times g \times \rho_U" + "\left(H_B - \Delta h_B\right) \times g \times \rho_B"$$

Maximum Rising Height of Bottom Layer



>Hydrostatic Pressure on the Bottom Surface Must be Uniform.

$$P_C = P_B$$

Substituting Explicit Form

$$H_U g\rho_U + H_B g\rho_B = \left(H_U + \Delta h_U + \Delta h_B\right) g\rho_U + \left(H_B - \Delta h_B\right) g\rho_B$$

$$H_U \rho_U + H_B \rho_B = \left(H_U + \Delta h_U + \Delta h_B\right) \rho_U + \left(H_B - \Delta h_B\right) \rho_B$$

Divided by g

Maximum Rising Height of Bottom Layer

$$H_U \rho_U + H_B \rho_B = \left(H_U + \Delta h_U + \Delta h_B\right)\rho_U + \left(H_B - \Delta h_B\right)\rho_B$$

Simplifying

$$\left(\rho_B - \rho_U\right)\Delta h_B = \Delta h_U \rho_U$$

Solve for Δh_B

$$\therefore \Delta h_B = \frac{\rho_U}{\rho_B - \rho_U}\Delta h_U$$

Density of Water is Large; $\rho_U \cong 1{,}000[kg/m^3]$

Ordinary, Difference of Density $\left(\rho_B - \rho_U\right)$ is small,

eg). In the Thermal Stratification in Lakes; $\Delta T \cong 10$ [℃] ⇨ $\left(\rho_B - \rho_U\right) \cong 2.0[kg/m^3]$

Rising Height of Interface; $\Delta h_B = \frac{\rho_U}{\rho_B - \rho_U}\Delta h_U = \frac{1{,}000}{2} \cong 1{,}000 \times \Delta h_U$

※ Rising Height of Stratification Reaches Up to 1,000 times Larger than Water Surface!

Maximum Rising Height of Bottom Layer

Typical Scale of Lakes,

Width: $L \approx 10km$

Depth: $H \approx 10m$

Wind Speed : 10m/s

$$\left.\begin{array}{l} L \approx 10km \\ H \approx 10m \\ \text{Wind Speed : 10m/s} \end{array}\right\} \Delta h_U = 6.5[cm]$$

Rising Height of Interface;

$$\Delta h_B \cong 1{,}000 \times \Delta h_U = 10^3 \times 6.5 \times 10^{-2} = 65[m]$$

> Even if there is Small Fluctuation in Water Surface,
Interface of Stratification Changes Very-Very Drastically!

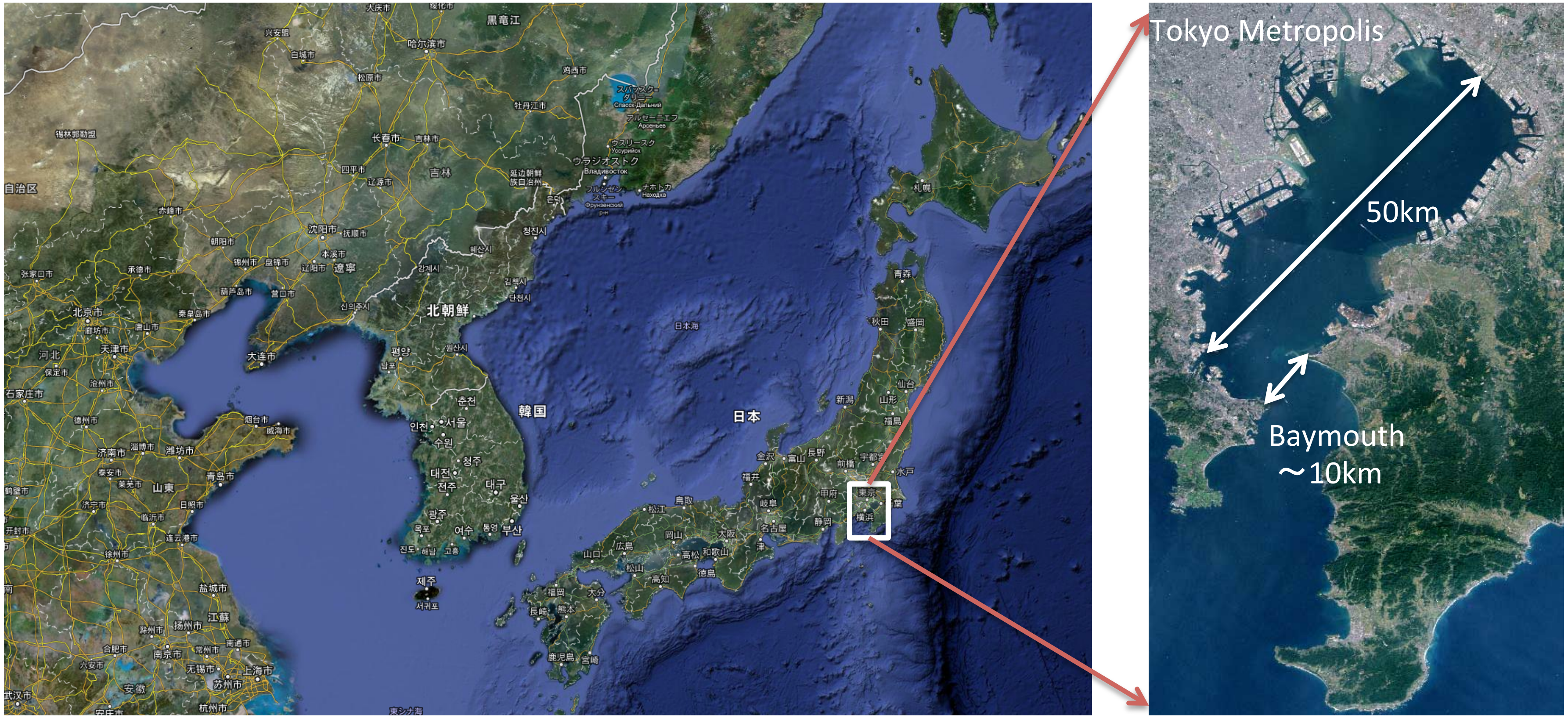
>This Extreme Rising Height of Interface Often
Results in Sevier Environmental Impacts.

Environmental Impact of Upwelling in Tokyo Bay

>Tokyo Bay Adjoins the Tokyo Metropolis

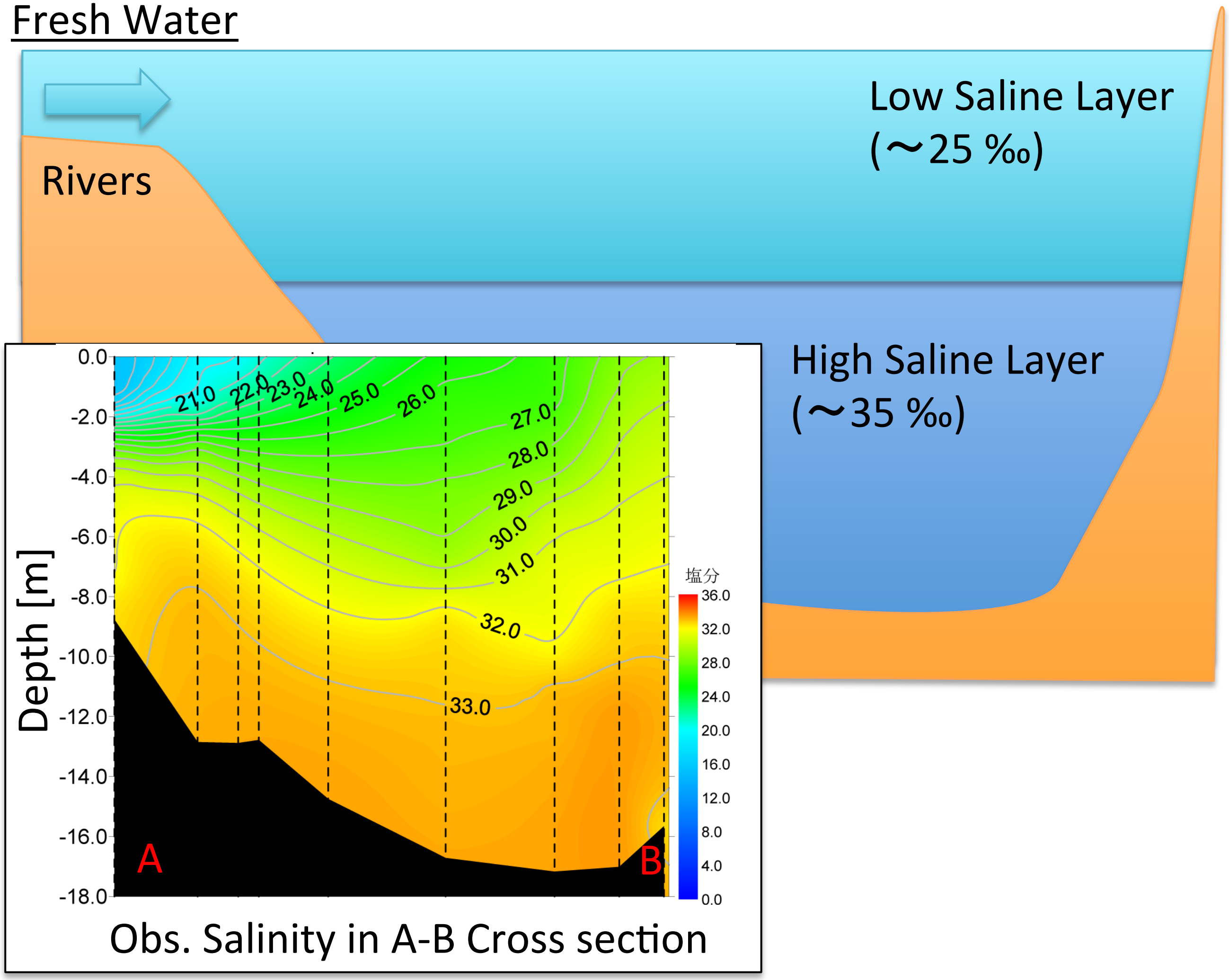
>Width ～ 50km

>Pinched Baymouth ～ 10km → Semi-Closed Water Area.



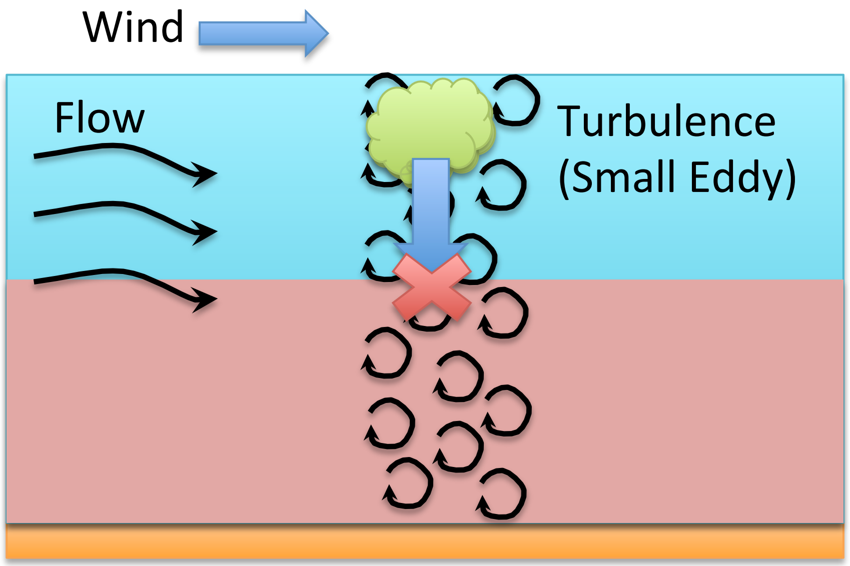
Environmental Impact of Upwelling in Tokyo Bay

> Several Rivers Emptying to Tokyo Bay.
> Fresh Water Tends to Stay in the Bay Due to Pinched Baymouth.
> Salinity Stratification is Observed through the Year.



Obs. Salinity in A-B Cross section

Vertical Transportation in Actual Water Environments

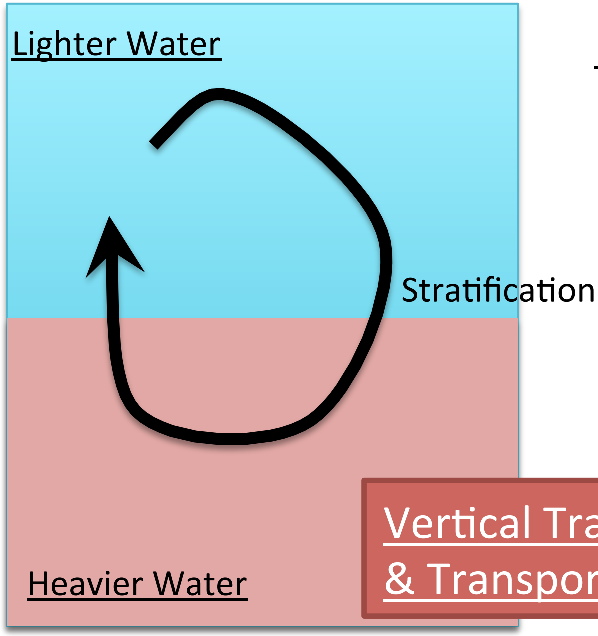


In Actual Water Environments,

Due to the Flows, Wind, Frictions,…

Turbulence is Generated.
Turbulence Consists of Small Eddies.

Due to the Eddies,
Substances (Such as DO) are Transported in Vertical Direction.



If there is Density Stratification,

Eddy Existing Around the Interface
Must Lift the Heavier Water Upward.

Energy of Eddy is Consumed
&
Eddy Around the Interface is Dismissed.

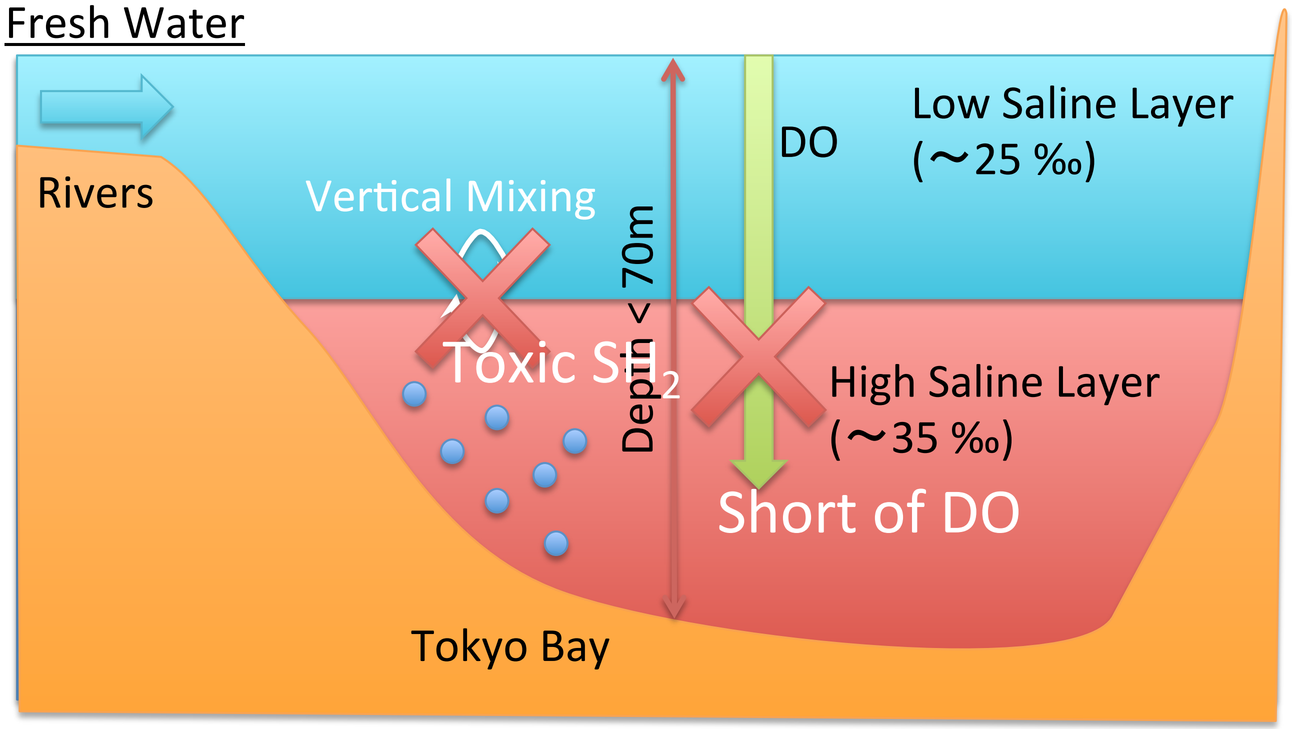
Vertical Transportation Across the Interface is Obstructed
& Transport of Substances from Upper to Bottom is Suppressed.

Environmental Impact of Upwelling in Tokyo Bay

> Depth Reach to 70m (Deep Water Area).

> Under Stratification, Vertical Mixing is Generally Suppressed.

> Dissolved Oxygen (DO) can not Supply to Bottom Water Layer.



> Bottom Water Layer is Kept to be Short of DO & Anaerobic.

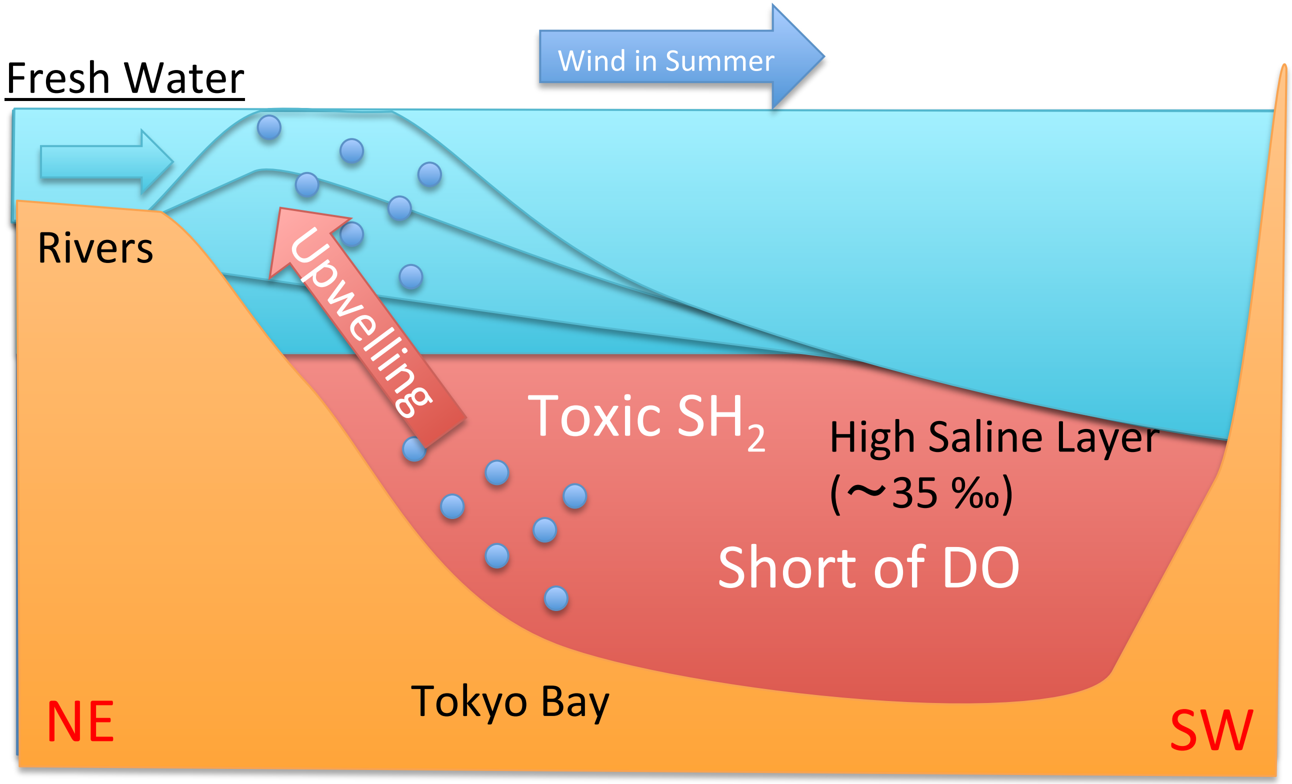
> Under Anaerobic Condition, Some Bacteria Products Toxic hydrogen sulfide (SH_2).

- Trough the year, Bottom Water Layer is Kept to be Bad Water Quality.
- Aquatic Life (Fish, Shellfish) can not Survive in the Bottom Water.

Environmental Impact of Upwelling in Tokyo Bay

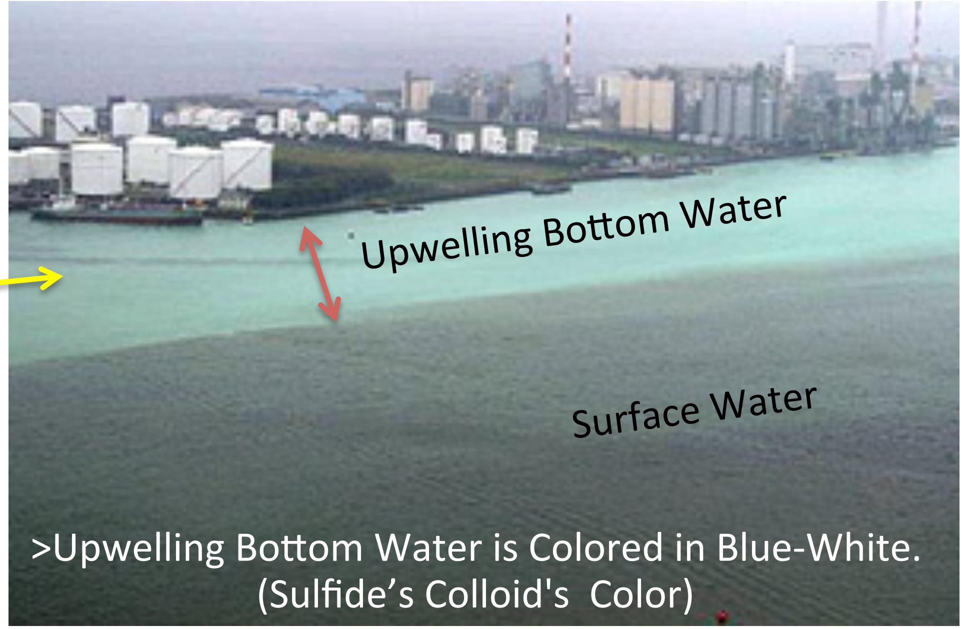
> Strong Wind Directing to South-West Often Blows in Summer.

> Upwelling of Toxic Bottom Water Occurs in Downwind Area (NE).



> Toxic Water Reaches to Surface Regions in NE Area.

Chiba Port



>Upwelling Bottom Water is Colored in Blue-White.
(Sulfide’s Colloid's Color)

> Upwelling Toxic & Hypoxic Water
Causes Massive Deaths of Fishes.

Summary of Feature of Density Stratification in Steady State

> Interface of Stratification (Thermocline & Halocline) is not Always Flat.

> Interface Changes According to the Change of Water Surface.

> When the Water Surface Inclines (eg. Wind Set-Up),
Interface also Inclines and Upwelling flow can be Generated around the Edge.

> Rising Height of Interface of Stratification Tends to be Rather Larger than that of the Water Surface, and Can Reach to Several tens meter in Actual Water Environments.

> By Upwelling Flow, In Some Cases, Bottom Toxic (Bad Water Quality) Water Can Rise to Surface Shallow Area. As a Result, Sevier Disasters (Massive Deaths of Fishes) Can be Caused.